

## Amendments to the Claims

This listing of claims will replace all prior versions, and listings, of claims in the application.

1. (currently amended) A method of fragile watermarking, ~~characterised by the step of comprising:~~  
generating at least a first ill-conditioned operator, ~~said ill-conditioned operator being~~  
related to values extracted from an image or portion thereof  $A$ ; and  
replacing a non-zero singular value of a singular value matrix  $S_A$  of an image or portion thereof  $A$ , with a solution to a linear equation comprising the ill-conditioned operator, wherein the non-zero singular value to be replaced is the smallest non-zero singular value  $S_r(A)$  in a singular value matrix  $S_A$  of rank  $r$ .
2. (original) A method of fragile watermarking according to claim 1 wherein the ill-conditioned operator is generated by altering a value to increase the operator's condition number.
3. (canceled)
4. (canceled)
5. (previously presented) A method of fragile watermarking according to claim 1, wherein a non-zero singular value of a singular value matrix  $S_W$  of a watermark pattern or portion thereof  $W$  is replaced, such that said replacement increases the condition number of the singular value matrix  $S_W$  of the watermark pattern or portion thereof  $W$ , wherein the non-zero singular value to be replaced is the smallest non-zero singular value  $S_t(W)$  in a singular value matrix  $S_W$  of rank  $t$ .
6. (canceled)

7. (previously presented) A method of fragile watermarking according to claim 5, wherein a replacement non-zero singular value of singular value matrix  $S_W$  of a watermark or portion thereof  $W$  is calculated by calculating substantially the following equation part:

$$S_{\varepsilon}(\hat{W}) = \varepsilon,$$

where  $\varepsilon$  is a small positive real number that increases the condition number of the singular value matrix  $S_W$ .

8. (previously presented) A method of fragile watermarking according to claim 1, wherein the step of generating at least a first ill-conditioned operator comprises calculating substantially the following equation part:

$$B = \hat{A}\hat{W},$$

where  $\hat{W}$  is substantially constructed according to  $\hat{W} = U_w \hat{S}_w V_w^T$ ,  $\hat{S}_w$  comprising at least one altered singular value  $S_{\varepsilon}(\hat{W}) = \varepsilon$ , and such that  $B$  forms a parametric family of matrices

$$B(\hat{S}_{\varepsilon}) = \hat{A}(\hat{S}_{\varepsilon})\hat{W} \text{ for possible values of } \hat{S}_{\varepsilon}(A).$$

9. (previously presented) A method of fragile watermarking according to claim 8, wherein  $\hat{S}_{\varepsilon}(A)$  is determined by an  $L_2$ -norm solution of the least squares problem  $\min_{x \in \mathfrak{R}^P} \|Bx - b\|_2^2$  to equal the square of a predefined key  $N$  of predetermined value, where  $b$  is an arbitrary vector.

10. (previously presented) A method of fragile watermarking according to claim 3, wherein the replacement non-zero singular value of singular value matrix  $A$  is calculated by calculating substantially the following equation part:

$$\min_{\hat{S}_{\varepsilon}(A)} \left\{ \sum_{i=1}^{\alpha} \left( u_{B_i}^T b / S_i(B(\hat{S}_{\varepsilon})) \right)^2 - N^2 \right\},$$

where  $u_{B_i}$  is the  $i$ -th column of the matrix formed with the right singular vectors of  $B$ .

11. (original) A method of fragile watermarking according to claim 10, wherein  $\hat{S}_r(A)$  further satisfies

$\hat{S}_r(A) = \overline{S}_r(A) \in [\max(e\text{ps}, S_r(A) - \delta), S_r(A) + \delta] = [H_0, H_1]$ , where  $\delta$  is a distortion control and  $e\text{ps}$  is machine precision, such that the step of calculating the replacement non-zero singular value comprises calculating substantially the following equation part:

$$\hat{S}_r \in [H_0, H_1] \left\{ \sum_{i=1}^q (u_{B_i}^T b / S_i(B(\hat{S}_r)))^p - N^2 \right\},$$

with all terms as defined herein.

12. (canceled)

13. (currently amended) A method of fragile watermarking according to ~~claim 12~~claim 11, wherein for a sequential watermarking process comprising the watermarking of portion  $A^{(k)}$  after the watermarking of portion  $A^{(k-1)}$ ,  $k=1, \dots, L$  of  $L$  portions, then the step of calculating  $b^{(k)}$  for portion  $A^{(k)}$  comprises calculating substantially the following equation part:

$$b^{(k)} = \begin{cases} A^{(k)} Z^{(k)} & \text{for } k = 1 \\ A^{(k-1)} Z^{(k)} & \text{else} \end{cases},$$

where  $Z(k)$  is a pseudo-random binary vector.

14. (previously presented) A method of fragile watermarking according to claim 1, wherein a watermarked image or portion thereof  $\hat{A}$  comprises calculating substantially the following equation part:

$$\hat{A} = U_A \hat{S}_A V_A^T$$

where  $\hat{S}_A$  comprises at least one replaced singular value,  $U_A$  and  $V_A$  being left and right singular matrices.

15. (previously presented) A method of fragile watermarking according to claim 1, wherein a watermark pattern or portion thereof  $W$  is generated by a pseudo-random generator seeded by a key  $K$  of predetermined value.

16. (canceled)

17. (previously presented) A method of fragile watermarking according to claim 15, wherein the-a watermark pattern or portion thereof  $W$  is generated by a pseudo-random generator seeded by a key  $K$  of predetermined value, combined with either a single or repeated instance of a logo.

18. (previously presented) A method of fragile watermarking according to claim 1, comprising the following steps;

- i. generating a  $K$ -dependent watermark pattern  $W$  from  $\Omega$ , or recalling a pre-existing one;
- ii. constructing a parametric family of matrices  $B(\hat{S}_r)$ ;
- iii. estimating a unique parameter  $\bar{S}_r(A)$ , that minimizes the expression

$$\min_{\hat{S}_r} \left\{ \sum_{i=1}^n \left( u_{B_i}^T b / s_i(B(\hat{S}_r)) \right)^2 - N^2 \right\}; \text{ and}$$

- iv. estimating the watermarked block  $\hat{A} = U_A \hat{S}_A V_A^T$  by setting

$$\hat{S} = \text{diag}(s_1(A), \dots, s_{r-1}(A), \bar{S}_r(A)).$$

19. (previously presented) A method of fragile watermarking according to claim 1, comprising the following steps;

- i. generating a  $K$ -dependent watermark pattern  $W$  from  $\Omega$ , or recalling a pre-existing one;
- ii. constructing a parametric family of matrices  $B(\hat{S}_r)$ ;
- iii. estimating a unique parameter  $\bar{S}_r(A) \in [\max(e\psi s, s_r(A) - \delta), s_r(A) + \delta] = [H_0, H_1]$ , that minimizes the expression:

$$\min_{\hat{S}_r \in [H_0, H_1]} \left\{ \sum_{i=1}^q \left( u_{\beta_i}^T b / s_i(B(\hat{S}_r)) \right)^p - N^2 \right\}; \text{ and}$$

- iv. estimating the watermarked block  $\hat{A} = U_A \hat{S}_A V_A^T$  by setting

$$\hat{S} = \text{diag}(s_1(A), \dots, s_{r-1}(A), \bar{S}_r(A)).$$

20. (currently amended) A method of verifying a fragile watermark, characterised by the ~~step of comprising:~~

generating at least a first ill-conditioned operator by altering a value to increase its condition number, said ill-conditioned operator being related to values extracted from a received image or portion thereof  $A^*$ ; and

calculating a solution to the least squares problem  $\min_{x \in \mathfrak{R}^p} \|B^*x - b\|_2^2$  where

$$\underline{B^* = A^* \hat{W}}.$$

21. (canceled)

22. (previously presented) A method of verifying a fragile watermark according to claim 20, wherein a positive square-root  $N^*$  of the  $L_2$ -norm solution of the least squares problem

$$\min_{x \in \mathbb{R}^p} \|B^*x - b\|_2^2 \text{ is compared with key } N; \text{ and}$$

the received image or portion thereof  $A^*$  comprising the fragile watermark is declared authentic if  $|N^* - N| \leq \tau$ , where  $\tau$  is a threshold value.

23. (previously presented) A method of verifying a fragile watermark according to claim 22, wherein the value  $N^*$  is calculated by calculating substantially the following equation part:

$$(N^*)^2 = \sum_{i=1}^n \left( u_{B_i}^T b / s_i(B^*) \right)^2;$$

$N^*$  is compared with key  $N$ ; and

the received image or portion thereof  $A^*$  comprising the fragile watermark is declared authentic if  $|N^* - N| \leq \tau$ , where  $\tau$  is a threshold value.

24. (canceled)

25. (canceled)